1-1-2009

Review: Spectra of Toeplitz Operators and Compositions of Muckenhoupt Weights with Blaschke Products

Stephan Ramon Garcia
Pomona College

Recommended Citation

MR2414440 (2009c:47034) Grudsky, S., Shargorodsky, E., Spectra of Toeplitz operators and compositions of Muckenhoupt weights with Blaschke products, Integral Equations Operator Theory 61 (2008), no. 1, 63–75. (Reviewer: Stephan R. Garcia)

MR2414440 (2009c:47034) 47B35 (30D50 45P05 47A10 47B38)
Grudsky, Sergei [Grudskiĭ, Sergeĭ M.] (MEX-IPN-CI);
Shargorodsky, Eugene [Shargorodskiĭ, E. M.] (4-LNDKC)
Spectra of Toeplitz operators and compositions of Muckenhoupt weights with Blaschke products. (English summary)
Integral Equations Operator Theory **61** (2008), *no. 1,* 63–75.

The authors discuss the optimality of a sufficient condition from [E. M. Shargorodskiĭ, Integral Equations Operator Theory **57** (2007), no. 1, 127–132; MR2294278 (2007m:47063)] for a point $\lambda\in\mathbb{C}\setminus a(\mathbb{T})$ to belong to the essential spectrum of a Toeplitz operator $T(a)$ with symbol $a\in L^\infty(\mathbb{T})$. In particular, they provide a negative answer to a question raised in [E. M. Shargorodskiĭ, Integral Equations Operator Theory **20** (1994), no. 1, 119–123; MR1290460 (95h:45013)].

To be more specific, let $\mathbb{T}$ denote the unit circle and recall that $c\in\mathbb{C}$ is called a cluster value (resp. left cluster value, resp. right cluster value) of a measurable function $a\colon\mathbb{T}\to\mathbb{C}$ at a point $\zeta\in\mathbb{T}$ if the function $1/(a-c)$ does not belong to $L^\infty(W)$ for every neighborhood (resp. left semi-neighborhood, resp. right semi-neighborhood) $W\subset\mathbb{T}$ of ζ. The set of all left (resp. right) cluster values of a at ζ is denoted by $a(\zeta-0)$ (resp. $a(\zeta+0)$). The authors prove the following theorem:

Theorem 1.2. There exists $a\in L^\infty(\mathbb{T})$ such that $a(1-0)=\{-1,+1\}$, $|a|=1$, $T(a)\colon H^p(\mathbb{T})\to H^p(\mathbb{T})$ is invertible for any $p\in(1,2)$, and $T(1/a)\colon H^p(\mathbb{T})\to H^p(\mathbb{T})$ is invertible for any $p\in(2,+\infty)$.

The approach is based on a new sufficient condition (Theorem 1.3) for the composition of a Muckenhoupt weight with a Blaschke product to belong to the same Muckenhoupt class.

Reviewed by *Stephan R. Garcia*

References

1. A. Böttcher and S. Grudsky, *On the composition of Muckenhoupt weights and inner functions.* J. Lond. Math. Soc., II. Ser. **58,** No.1 (1998), 172–184. MR1666110 (99j:47033)
2. A. Böttcher and B. Silbermann, *Analysis of Toeplitz Operators.* Springer-Verlag, 1990. MR1071374 (92e:47001)
3. K.F. Clancey, *One dimensional singular integral operators on L^p.* J. Math. Anal. Appl. **54** (1976), 522–529. MR0415229 (54 #3320)
4. K.F. Clancey, *Corrigendum for the article "One dimensional singular integral operators on L^p"*. J. Math. Anal. Appl. **99** (1984), 527–529. MR0737821 (85b:45005)
5. P.L. Duren, *Theory of H^p Spaces.* Academic Press, 1970, and Dover, 2000. MR0268655 (42 #3552)
6. V. Dybin and S.M. Grudsky, *Introduction to the theory of Toeplitz operators with infinite index.* Birkhäuser Verlag, 2002. MR1942445 (2003j:47031)
7. J.B. Garnett, *Bounded Analytic Functions.* Academic Press, 1981. MR0628971 (83g:30037)
8. I. Gohberg and N. Krupnik, *One-Dimensional Linear Singular Integral Equations I & II.*

Birkhäuser Verlag, 1992. MR1138208 (93c:47061)
9. S.M. Grudskij and A.B. Khevelev, *On invertibility in $L^2(R)$ of singular integral operators with periodic coefficients and a shift*. Sov. Math. Dokl. **27** (1983), 486–489.
10. E. Shargorodsky, *On singular integral operators with coefficients from $P_n\mathbb{C}$*. Tr. Tbilis. Mat. Inst. Razmadze **93** (1990), 52–66 (Russian). MR1119057 (92e:47043)
11. E. Shargorodsky, *On some geometric conditions of Fredholmity of one-dimensional singular integral operators*. Integral Equations Oper. Theory **20,** No.1 (1994), 119–123. MR1290460 (95h:45013)
12. E. Shargorodsky, *A remark on the essential spectra of Toeplitz operators with bounded measurable coefficients*. Integr. Equ. Oper. Theory **57** (2007), 127–132. MR2294278 (2007m:47063)
13. I. M. Spitkovskij, *Factorization of matrix-functions belonging to the classes $A_n(p)$ and TL*. Ukr. Math. J. **35** (1983), 383–388 (translation from Ukr. Mat. Zh. **35,** No.4 (1983), 455–460). MR0712465 (84k:30045)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.